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# FINAL REPORT #N00014-92-J-1876

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The ovevall objective of this work is to analyze and design effective computational algorithms for the integration of evolution equations over long time intervals. Many models of physical significance are characterised by the property of "sensitive dependence on initial conditions": small changes in the given data can make large changes in the detailed output of the model. Examples of such systems include weather or climate models in certain parameter regimes and turbulent flow problems. For such systems the effect of numerical approximation is not immediately clear. We may view numerical approximation as a small perturbation and the previous discussion indicates that this can nonetheless have a large effect on the detailed output from the model, over long time intervals. Thus it is important to know how to *interpret data from such numerical simulations.* Furthermore, in long-time integration, it is often crucial that the correct energy balance be used in the equation – be it dissipation or conservation. Thus it is important to design methods which *replicate the energy balance in the equation* under mild or no restrictions on the discretization parameters.

These objective have been acheieved and the following list of Awards, Invited Presentations, Graduated Students and Publications are all directly related to the support obtained through this grant.

## Awards

**Monroe Martin Prize in Applied Mathematics; Joint 1st Prize. February 1996**

Awarded every five years by the Institute of Physical Sciences and Technology, University of Maryland, to an individual under the age of 35, for an outstanding paper in applied mathematics. See [5b].

**James Wilkinson Prize in Numerical Analysis and Scientific Computing. February 1997**

Awarded every six years by SIAM (the Society for Industrial and Applied Mathematics) for research over the six years preceeding the award.

## Invited Presentations at Conferences

**LMS/SERC Symposium on Evolution Problems, University of Durham, England; July 1st–July 14th 1992.** *"Numerical Analysis of Dissipative and Gradient Dynamical Systems".*

**Canadian Applied Mathematics Society Meeting, York, Ontario; May 31st–June 3rd 1993.** *"Long Time Approximation Theory for Evolution Equations".*

**Symposium in Honour of Leslie Fox, Oxford, England; June 24th–June 25th 1993.** *"Analysis and Computations for a Model of Solid Phase Transitions".*

**Biennial Numerical Analysis Conference, Dundee, Scotland; June 29th–July 2nd 1993.** *"Approximation of Dissipative Partial Differential Equations Over Long Time Intervals".*

**$VI^{th}$ Science and Engineering Research Council Summer School in Numerical Analysis, University of Leicester, England; July 18th–July 29th 1994.** *Series of 5 Lectures on "Numerical Analysis of Dynamical Systems".*

**Canadian Applied Mathematics Society Meeting, St. John's, Newfoundland; May 31st– June 3rd 1995.** *"Deterministic and Probabilistic Analysis of Adaptive Algorithms for Initial-Value Problems".*

**NSF-CBMS Conference on Approximation Dynamics, Columbia, Missouri; June 1st–June 5th 1995.** *"Numerical Stability Issues in Long-Time Simulations."*

**ODE to NODE, Norwegian Workshop on Future Directions in Numerical Solution of Ordinary Differential Equations; June 19th–22nd.** *"Numerical Analysis and Dynamical Systems."*

**Conference on Dynamical Numerical Analysis, Atlanta, Georgia; December 14th–16th 1995.** *"Software for Initial-Value Problems as a Discontinuous Dynamical System."*

**The State of the Art in Numerical Analysis, University of York, England; April 1st–6th 1996.** *"Convergence and Stability in the Numerical Analysis of Dynamical Systems".*

**Invariant Measures and Invariant Manifolds; Workshop on Ergodic Theory and Numerical Analysis of Dynamical Systems. Brakel, Germany, June 2nd–5th 1996.** *"Probabilistic Techniques in the Numerical Analysis of Dynamical Systems."*

**Australia-New Zealand Applied Mathematicis Annual Meeting, Melbourne, Australia; February 2nd–7th 1997.** "Computational Aspects of Deterministic and Random Dynamical Systems".

**SIAM Annual Meeting, Stanford, July 14th–18th 1997.** "Numerical Algorithms as Dynamical Systems".

**SciCADE 97, Scientific Computing and Differential Equations, University of Trieste; September 15th–19th 1997.** "Perturbation theory for invariant measures of Markov chains".

## Research Supervision

- Gabriel Lord. PhD. (Currently an EPSRC post-doc in the Department of Engineering Mathematics at Bristol University.)
- Antony Humphries. PhD. (Currently a permanent faculty member in the Mathematics Department at Sussex University).
- Jeremy Smith. PhD. (Currently employed in a software company in silicone valley).
- Hamid Samandari. PhD. (Currently Employed by McKinsey).
- Tony Shardlow. PhD. (Takes up a Postdoctoral Position at IMA Minnesota in 9/97).

• Oscar Gonzalez. PhD. (Currently a post-doc at the Institute for Physical Sciences and Technology at the University of Maryland).
• Martin Gander. PhD. (Takes up a Postdoctoral Position at Ecole Polytechnique in 9/97).
• Fengshan Bai. Post-Doc. (Currently a permanent faculty member in Applied Mathematics at Tsinghua University, Beijing).

REFEREED PUBLICATIONS IN ARCHIVAL JOURNALS

[22] C.M.Elliott and A.M.Stuart, *The global dynamics of discrete semilinear parabolic equations.* SIAM J. Num. Anal. **30**(1993), 1622–1663.

[25] F.Bai, A. Spence and A.M.Stuart, *The Numerical Computation of Heteroclinic Connections in Systems of Gradient Partial Differential Equations.* SIAM J. Appl. Math. **35**(1993), 743–769.

[26] A.R. Humphries and A.M. Stuart *Runge-Kutta methods for disspative and gradient dynamical systems.* SIAM J. Num. Anal. **31**(1994), 1452–1485.

[27] A.M. Stuart and A.R. Humphries *Model Problems in Numerical Stability for Initial Value Problems.* SIAM Review **36**(1994), 226–257.

[28] F. Bai, A. Spence and A.M. Stuart *Numerical Computations of Coarsening in the Cahn-Hilliard model of phase separation.* Physica D. **78**(1994), 155–165.

[29] F. Bai, C.M. Elliott, A. Gardiner, A. Spence and A.M. Stuart *The viscous Cahn-Hilliard equation. Part I: Computations.* Nonlinearity **8**(1995), 131–160.

[30] D.J. Estep and A.M. Stuart *The rate of error growth in Hamiltonian conserving integrators.* ZAMP **46**(1995), 407–418.

[31] G.J. Lord and A.M. Stuart *Gevrey regularity and upper semicontinuity of attractors for a semi-discrete Ginzburg-Landau equation.* Num. Func. Anal. Appl. **16**(1995), 1003-1047.

[32] A.M. Stuart and A.R. Humphries *The Essential Stability of Local Error Control for Dissipative and Gradient Dynamical Systems.* SIAM J. Num. Anal. **32**(1995), 1940–1971.

[33] D.A. Jones and A.M. Stuart *Attractive Invariant Manifolds Under Approximation. Part I: Inertial Manifolds.* J. Diff. Eq. **123**(1995), 588–637.

[34] C.M. Elliott and A.M. Stuart *The viscous Cahn-Hilliard equation. Part II: Analysis.* J. Diff. Equations. **128**(1996), 387–414.

[35] M. Bjorhus and A.M. Stuart *Waveform relaxation as a dynamical system.* To appear Math. Comp.

[36] D.J. Higham and A.M. Stuart *Analysis of the dynamics of error control via a piecewise continuous residual.* Submitted to BIT.

[37] C.J. Budd, G.P. Koomullil and A.M. Stuart *On the solution of convection-diffusion boundary-value problems by grid adaptation.* To appear in SIAM J. Sci. Stat. Comp.

[38] A.M. Stuart *Probabilistic and Deterministic Convergence Proofs for Software for Initial Value Problems.* To appear Num. Alg.

[39] M. Gander and A. M. Stuart *Parallel Algorithms for the Solution of Time-Dependent Partial Differential Equations.* To appear in SIAM J. Sci. Comp.

[40] J. Smith and A.M. Stuart *Analysis of continuous moving mesh equations* Submitted to Applied Numerical Mathematics.

[41] D.A. Jones, A.M. Stuart and E.S. Titi *Persistence of Invariant Sets for Dissipative Evolution Equations.* Submitted to Math. Anal. and Applics.

OTHER PUBLICATIONS

[3b] A.R. Humphries, D.A. Jones and A.M. Stuart *Approximation of dissipative partial differential equations over long time intervals.* Appears in: "Numerical Analysis", editors G.A.Watson and D.F. Griffiths, Longman 1994.

[4b] A.M. Stuart *Numerical Analysis of Dynamical Systems.* Acta Numerica 1994, Cambridge University Press, Cambridge, 1994, pages 467–572.

[5b] A.M. Stuart *Perturbation Theory for Infinite Dimensional Dynamical Systems.* Appears in "Advances in Numerical Analysis", editor M. Ainsworth, J. Levesley, M. Marletta and W.A. Light, Oxford University Press, Oxford, 1995, 105 pages.

[6b] A.M. Stuart and A.R. Humphries *Dynamical Systems and Numerical Analysis* Cambridge University Press, 1996, 686 pages.

[7b] A.M. Stuart *Convergence and stability in the numerical approximation of dynamical systems.* To appear in "Start of the Art in Numerical Analysis 1996", editors A. Iserles and G.A. Watson, Oxford University Press.